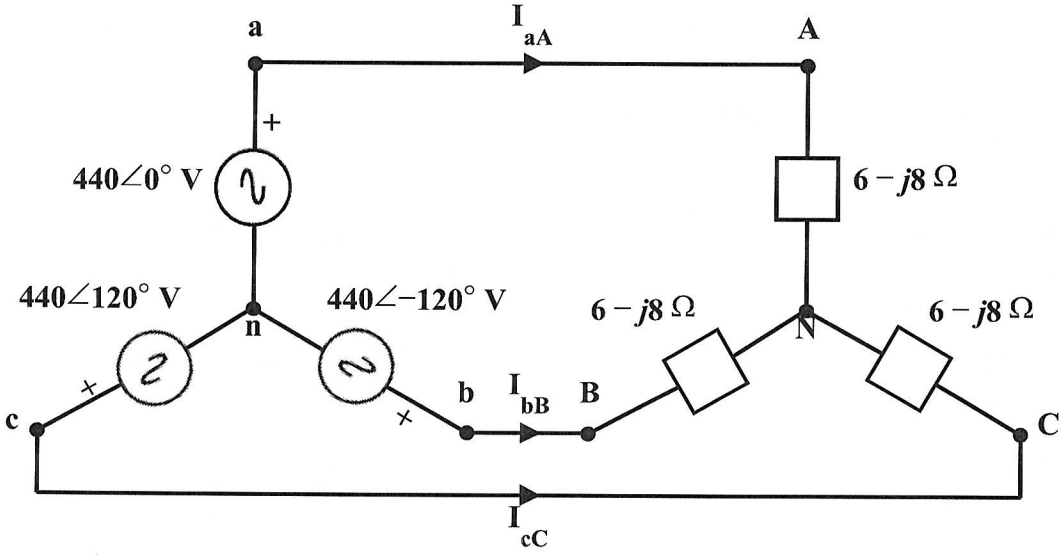
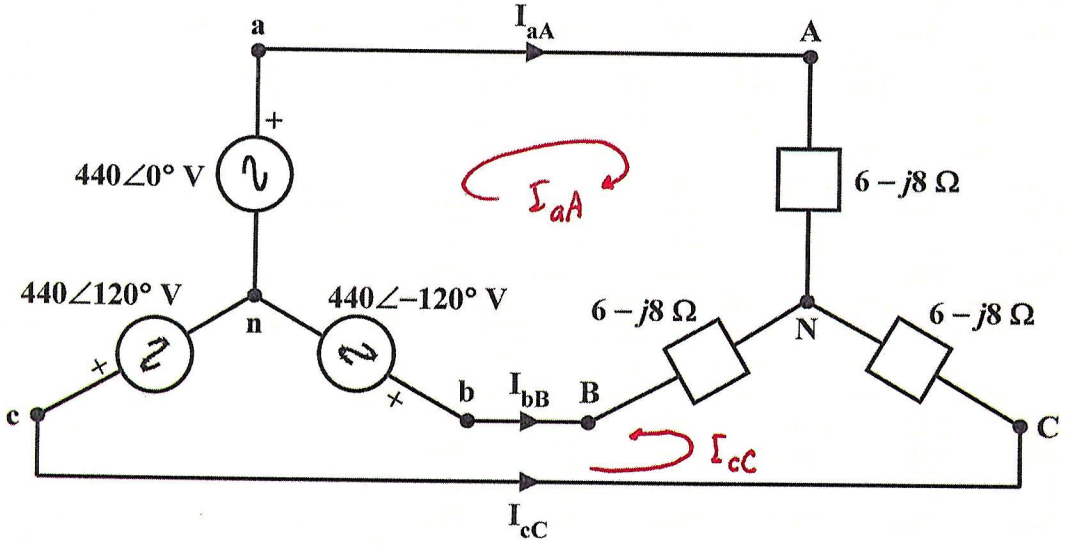
EE 3340

Homework Problem #042

Determine the line currents ($\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$ and $\mathbf{I}_{cC}$) in the three-phase balanced Y-Y circuit shown.



See the attached pages for 3 methods of solution.



$$-440\angle 0^\circ + (6-j8)\,\Sigma_{aA} + (6-j8)(\Sigma_{aA} + \Sigma_{cC}) + 440\angle -120^\circ = 0$$

$$440\angle 120^\circ - 440\angle -120^\circ - (6-j8)(\Sigma_{aA} + \Sigma_{cC}) - (6-j8)\,\Sigma_{cC} = 0$$

or

$$(12-j16)\,\Sigma_{aA} + (6-j8)\,\Sigma_{cC} = 440 - 440\angle -120^\circ$$
$$-(6-j8)\,\Sigma_{aA} - (12-j16)\,\Sigma_{cC} = 440\angle -120^\circ - 440\angle 120^\circ$$

or

$$(12-j16)\,\Sigma_{aA} + (6-j8)\,\Sigma_{cC} = 440\{1 - \cos d(120) - j * \sin d(-120)\}$$
$$-(6-j8)\,\Sigma_{aA} - (12-j16)\,\Sigma_{cC} = 440\{\cos d(-120) + j * \sin d(-120) - \cos d(120) - j * \sin d(120)\}$$

See MATLAB solution on the next page.

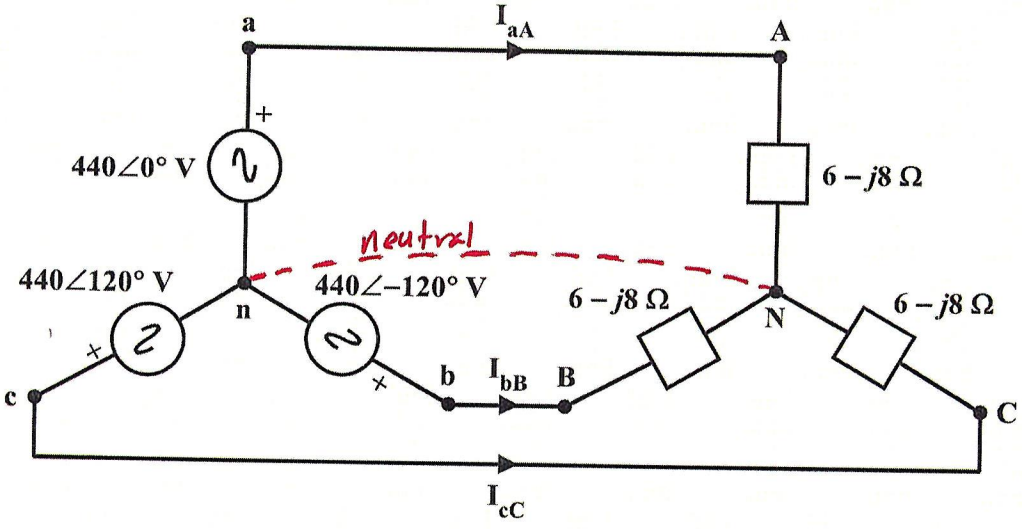
```
>> A=[12-j*16 6-j*8; -(6-j*8) -(12-j*16)]

A =
  12.0000 -16.0000i   6.0000 - 8.0000i
  -6.0000 + 8.0000i -12.0000 +16.0000i

>>   c=440*[1-cosd(-120)-j*sind(-120);   cosd(-120)+j*sind(-120)-
cosd(120)-j*sind(120)]

c =
   1.0e+02 *

   6.6000 + 3.8105i
   0.0000 - 7.6210i

>> meshCurrents=A\c

meshCurrents =
  26.4000 +35.2000i
 -43.6841 + 5.2631i

>> abs(meshCurrents(1))

ans =
    44

>> angle(meshCurrents(1))*180/pi

ans =
   53.1301

>> abs(meshCurrents(2))

ans =
   44.0000

>> angle(meshCurrents(2))*180/pi

ans =
  173.1301

>> IbB=-meshCurrents(1)-meshCurrents(2)

IbB =
  17.2841 -40.4631i

>> abs(IbB)

ans =
   44.0000

>> angle(IbB)*180/pi

ans =
  -66.8699
```

$I_{aA} = 44 \angle 53.13^\circ$ A

$I_{cC} = 44 \angle 173.13^\circ$ A

$I_{bB} = 44 \angle -66.87^\circ$ A

Alternately,

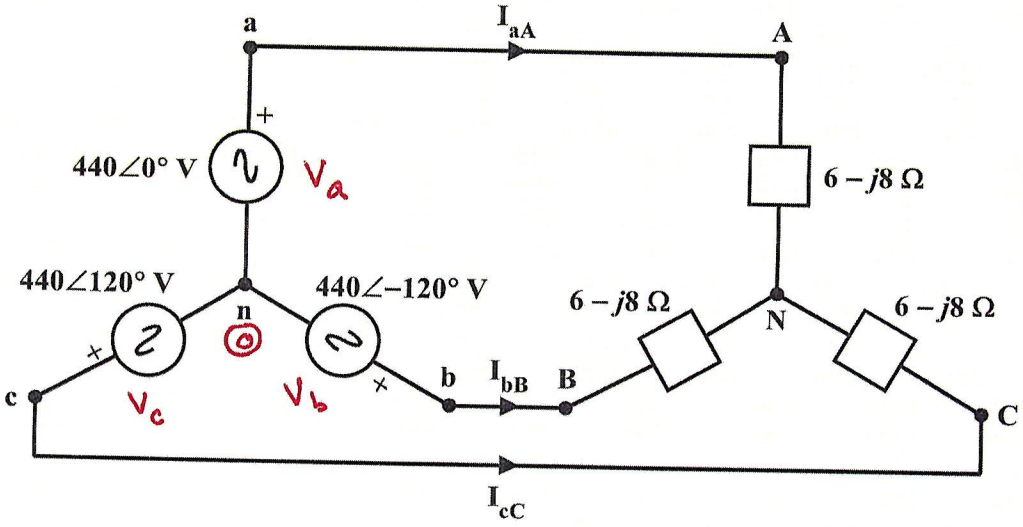


$$I_{aA} = \frac{440\angle 0^\circ}{6-j8} = 44\angle 53.13^\circ \text{ A}$$

$$I_{bB} = \frac{440\angle -120^\circ}{6-j8} = 44\angle -66.87^\circ \text{ A}$$

$$I_{cC} = \frac{440\angle 120^\circ}{6-j8} = 44\angle 173.13^\circ \text{ A}$$

Or



Use LTspice:

Assume $\omega = 1$ rad/s. Then

6 - j8 is a 6Ω resistor in series with a capacitor having value $\frac{1}{8}$ F.

```
Va   a   0   AC   440   0
Vb   b   0   AC   440   -120
Vc   c   0   AC   440   120
RA   A   1   6
CA   1   N   {1/8}
RB   B   2   6
CB   2   N   {1/8}
RC   C   3   6
CC   3   N   {1/8}
.AC  LIN  1   0.159154943   0.159154943
```

LTspice XVII - [04 .cir]

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```
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Va   a    0    AC   440 0
Vb   b    0    AC   440 -120
Vc   c    0    AC   440 120
RA   A    1    6
CA   1    N    {1/8}
RB   B    2    6
CB   2    N    {1/8}
RC   C    3    6
CC   3    N    {1/8}
.AC LIN 1    0.159154943 0.159154943
.end
```

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```
        --- AC Analysis ---

frequency:      0.159155        Hz
V(a):           mag:          440 phase: -3.70101e-015°      voltage
V(b):           mag:          440 phase:        -120°        voltage
V(c):           mag:          440 phase:         120°        voltage
V(1):           mag:          352 phase:    -36.8699°        voltage
V(n):           mag: 5.1238e-014 phase:     33.6901°        voltage
V(2):           mag:          352 phase:     -156.87°        voltage
V(3):           mag:          352 phase:     83.1301°        voltage
I(Cc):          mag:           44 phase:      173.13°        device_current
I(Cb):          mag:           44 phase:    -66.8699°        device_current
I(Ca):          mag:           44 phase:     53.1301°        device_current
I(Rc):          mag:           44 phase:      173.13°        device_current
I(Rb):          mag:           44 phase:    -66.8699°        device_current
I(Ra):          mag:           44 phase:     53.1301°        device_current
I(Vc):          mag:           44 phase:    -6.8699°         device_current
I(Vb):          mag:           44 phase:      113.13°        device_current
I(Va):          mag:           44 phase:     -126.87°        device_current
```

I_{cC} → I(Cc)

I_{bB} → I(Rb)

I_{aA} → I(Ra)

$I_{aA} = I(Ra) = 44 \angle 53.13°$ A

$I_{bB} = I(Rb) = 44 \angle -66.87°$ A

$I_{cC} = I(Rc) = 44 \angle 173.13°$ A